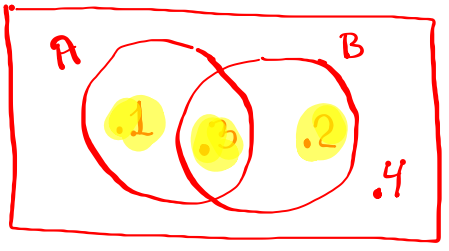
Math 227
Spring 2021
Lecture 14

Class QZ 16

$P(A) = .4$ $P(B) = .5$ $P(A \text{ and } B) = .3$

1) Venn Diagram



2) $P(A|B)$

$= \frac{P(A \text{ and } B)}{P(B)} = \frac{.3}{.5} = .6$ ✓

3) $P(A \text{ or } B)$

$= P(A) + P(B) - P(A \text{ and } B)$

$= .4 + .5 - .3 = .6$ ✓

Random Variable x with prob. dist. $P(x)$:

SG 15-18

Data
- 1) Qualitative
  - Non-Numerical
- 2) Quantitative
  - Numerical
    - 1) Discrete
      - Countable
    - 2) Continuous
      - Measureable

Let x be a discrete random variable with Prob. dist. $P(x)$. ↳ Countable

1) $0 \le P(x) \le 1$

2) $\sum P(x) = 1$

3) $P(x) = 1 \iff$ Sure event

4) $P(x) = 0 \iff$ Impossible event

5) $0 < P(x) \le .05 \iff$ Rare event

Prob. dist $P(x)$ gives prob. of all possible outcomes.

Prob. dist can be in the form of

1) table or chart

2) a formula

3) a graph

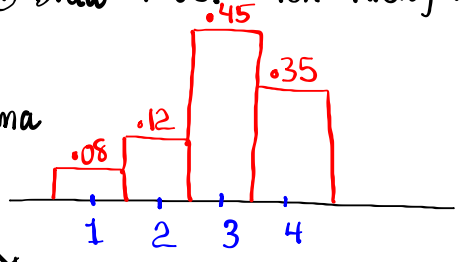
Consider the chart below

x	$P(x)$
1	.08
2	.12
3	.45
4	.35

① Verify $\sum P(x) = 1$

$.08 + .12 + .45 + .35 = 1$

② Draw Prob. dist. histogram



Mean μ ← mu

Variance σ^2 ← Sigma

Standard Deviation σ

$\mu = \sum x p(x)$ $\quad \sigma^2 = \sum x^2 p(x) - \mu^2$ $\quad \sigma = \sqrt{\sigma^2}$

How to find μ, σ, and σ^2 using TI:

$x \rightarrow$ L1 , $P(x) \rightarrow$ L2

STAT CALC 1-Var Stats

$\mu = \bar{x} = 3.07$

$\sigma = \sigma_x = .886$

$n = 1$

for σ^2 exact:

VARS 5: 4: x^2

with Menu:
List: L1
FreqList: L2
Calculate

x	P(x)
1	.08
2	.12
3	.45
4	.35

MATH 1: Enter

$\sigma^2 = .7851$

$\frac{7851}{10000}$

No Menu
L1, L2
Enter

Consider the chart below:

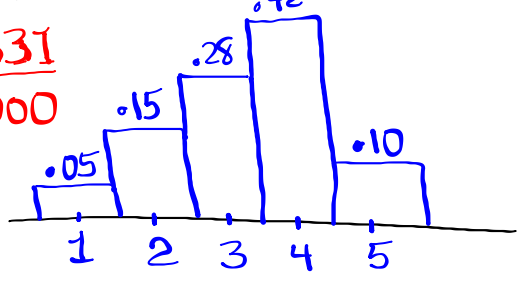
x	P(x)
1	.05
2	.15
3	.28
4	.42
5	.10

1) Find $P(x=5)$

$= 1 - [.05 + .15 + .28 + .42]$

$= .1$

2) Draw Prob. dist. histogram



$x \rightarrow$ L1 , $P(x) \rightarrow$ L2

3) Find

$\mu = \bar{x} = 3.37$

$\sigma = \sigma_x = 1.016$

σ^2 (exact) $= 1.0331$

$\frac{10331}{10000}$

A loaded die has the following prob.

$P(x=1)=.1$, $P(x=2)=.15$, $P(x=3)=.2$

$P(x=4)=.25$, $P(x=5)=.2$

1) Find $P(x=6)$

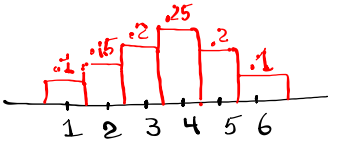
$P(x=6)=1-[.1+.15+.2+.25+.2]$

$=\boxed{.1}$

2) Complete this chart

x	$P(x)$
1	.1
2	.15
3	.2
4	.25
5	.2
6	.1

3) Draw Prob. dist. histogram



4) Find

$\mu=3.6$

$\sigma=1.463$

σ^2(exact) $=\frac{107}{50}$

68% Range $\Rightarrow \mu \pm \sigma$

Usual Range $\Rightarrow \mu \pm 2\sigma$

$\mu \approx 4$ 68% Range $\Rightarrow 4 \pm 1 \Rightarrow 3$ to 5

$\sigma \approx 1$ Usual Range $\Rightarrow 4 \pm 2(1) \Rightarrow 2$ to 6

A box has 2 dimes and 3 nickels.

Randomly take two coins, No replacement

NN $\rightarrow$ 10¢ $P(10¢)=\frac{3}{5}\cdot\frac{2}{4}=\frac{6}{20}=.3$

ND, DN $\rightarrow$ 15¢ $P(15¢)=$ $=\boxed{.6}$

DD $\rightarrow$ 20¢ $P(20¢)=\frac{2}{5}\cdot\frac{1}{4}=\frac{2}{20}=.1$

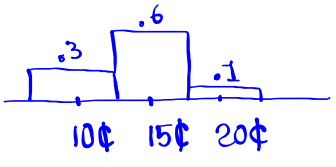
Total ¢	P(total ¢)
10¢	.3
15¢	.6
20¢	.1

Total ¢ $\rightarrow L1$ P(Total ¢) $\rightarrow L2$

$\mu=14$ $\sigma=3$ $\sigma^2=9$

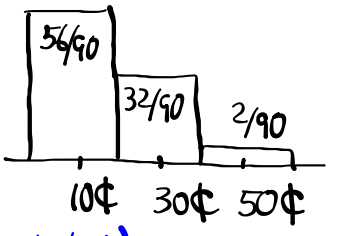
68% Range $\rightarrow \mu \pm \sigma \Rightarrow$ 11 to 17

Usual Range $\rightarrow \mu \pm 2\sigma \Rightarrow$ 8 to 20



2 Quarters, 8 Nickels

Select 2 Coins, No replacement



NN → 10¢

NQ
QN → 30¢

QQ → 50¢

Total ¢	P(Total ¢)	
10¢	$\frac{8}{10}\cdot\frac{7}{9}=\frac{56}{90}$	L1
30¢	$=\frac{32}{90}$	L1
50¢	$\frac{2}{10}\cdot\frac{1}{9}=\frac{2}{90}$	L1, L2

Use L1 & L2 to find

$\mu = 18$ $\sigma = 10.667$ $\sigma^2 = \frac{1024}{9}$

$\frac{8}{10}\cdot\frac{2}{9} + \frac{2}{10}\cdot\frac{8}{9} = \frac{32}{90}$...

See notes from Monday & Tuesday.

Class QZ 17

Given the chart below

x	P(x)
1	.12
3	.28
5	.25
7	.35

1) Find $P(x=5)$

$= 1 - [.12 + .28 + .35] = .25$

2) Use x & P(x) in L1 & L2 to find

$\mu = 4.66$ $\sigma = 2.080$ σ^2 (inexact)

$\frac{10811}{2500} =$

4.3244

Application

Expected Value $= \mu = \bar{x}$

I sold 50 TKts for \$10 each.

Draw one ticket $\Rightarrow$ winner gets a calc worth \$125. Expected Value / TKt sold.

Net gain	P(Net gain)
10 − 125	$\frac{1}{50}$
10 − 0	$\frac{49}{50}$

Net gain → L1

P(Net gain) → L2

Use L1 & L2 to find $\bar{x} = \mu =$ E.V.

Expected Value / TKT

\$1.50

You buy insurance for your luggage for \$100.

Any damages, Insurance Co. pays \$5000.

Prob. of any damage is .2%.

Find Expected Value per policy sold.

Net gain	P(Net gain)
100 − 5000	.2% = .002
100 − 0	99.8% = .998

Net gain → L1

Damage P(Net gain) → L2

No damage E.V. $= \mu = \bar{x}$

\$90

SG 15 & 16 ✓

Binomial Prob. Dist S& I7

1) n independent events

2) Each event has two outcomes.

$P(\text{Success}) = p$ $P(\text{Failure}) = q$

3) p & q remain unchanged for all n events. $p+q=1$, $q=1-p$

4) x is # of Successes

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

Consider a binomial prob. dist with $n=10$, $p=.6$

Find $P(x=7) = {}_{10}C_7 \cdot (.6)^7 \cdot (.4)^3$

$${}_nC_x \cdot p^x \cdot q^{n-x} = .215$$

$n=10$, $x=7$, $p=.6$, $q=.4$, $n-x=3$

10 MATH PRB nCr 7 * .6 ^ 7 * .4 ^ 3 Enter

A true-false exam has 12 questions.

You are making random guesses.

P(exactly 8 correct ans)

$n = 12$, $p = .5$, $q = .5$, $x = 8$ Successes

$n - x = 4$ Failures

$$P(x = 8) = {}_{12}C_8 \cdot (.5)^8 \cdot (.5)^4 = \boxed{.121}$$

Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard Deviation $\sigma = \sqrt{\sigma^2}$

} Binomial Prob. Dist.

Consider a binomial prob. dist with $n = 250$, and $p = .8$

$q = 1 - p = \boxed{.2}$ $\mu = np = \boxed{200}$

$\sigma^2 = npq = \boxed{40}$ $\sigma = \sqrt{\sigma^2} \approx \boxed{6.325}$

You are making random guesses on a multiple-choice exam with 400 questions. Each question has 5 choices but only one correct choice.

$n = 400$ $\quad p = \frac{1}{5} = .2$ $\quad q = \frac{4}{5} = .8$

$\mu = np = 400(.2) = \boxed{80}$

$\sigma^2 = npq = 400(.2)(.8) = \boxed{64}$

$\sigma = \sqrt{\sigma^2} = \sqrt{64} = \boxed{8}$

68% Range $\Rightarrow \mu \pm \sigma \Rightarrow \boxed{72 \text{ to } 88}$

95% Range $\Rightarrow \mu \pm 2\sigma \Rightarrow \boxed{64 \text{ to } 96}$